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THE NAME BEHIND THE CONSERVATION LAWS

THE MATHEMATICIAN WHO COULD NOT BE HIRED

In 1915 David HILBERT and Felix Klein brought a young ALGEBRAIST from Erlangen to Göttingen to help with a puzzle in Einstein's new theory of GRAVITATION. She could not be hired. German universities required a HABILITATION, the second thesis and formal examination that granted the right to lecture, and the philosophical faculty refused to open it to a woman, arguing that soldiers returning from the front would be insulted by a female instructor. For four years her courses appeared in the catalogue under Hilbert's name, taught by her. The mathematics was hers.

SPELL Spell: **ALGEBRAIST**

ALGEBRAIST

SPELL Spell: **GRAVITATION**

GRAVITATION

KNOWN German universities of the time demanded a second thesis and a formal examination before a scholar was granted the right to lecture. What was that qualification called?

HABILITATION

KNOWN What was the last name of the Göttingen mathematician who, with Felix Klein, brought the young algebraist from Erlangen?

HILBERT

OPEN The faculty argued that returning soldiers would be insulted by a woman lecturing. What is the weakest reason you have ever heard for keeping someone out?

Open response - use practitioner judgment

SYMMETRY AS BOOKKEEPING

The problem she was handed concerned ENERGY in Einstein's theory, and her answer ran far past it. Physics states its laws through a VARIATIONAL principle: a system follows the path that extremizes the action, a single quantity accumulated along the whole trajectory. She proved that whenever the action is left unchanged by a continuous transformation, an INVARIANCE, a matching CONSERVATION law follows automatically. Shift the clock, and energy is conserved. Shift the origin in space, and MOMENTUM is conserved. Rotate the phase of a charged field, and CHARGE is conserved. SYMMETRY is not a hint about the laws; it is their bookkeeping.

SPELL Spell: **VARIATIONAL**

VARIATIONAL

SPELL Spell: **INVARIANCE**

INVARIANCE

KNOWN What is the term for the property that is not a hint about the laws of physics but their bookkeeping?

SYMMETRY

KNOWN What is the term for a law guaranteeing that a quantity stays fixed because a continuous transformation leaves the action unchanged?

CONSERVATION

SEMI_OPEN Name two of the quantities that a symmetry keeps conserved.

NEEDS 2 OF **ENERGY / MOMENTUM / CHARGE**

OPEN Shift the clock and energy is conserved; shift the origin and momentum is. What is something in your life that stays the same no matter where you start counting?

Open response - use practitioner judgment

KEEP THE AXIOMS

The theorem was, by her own reckoning, a detour. Her real program was abstract algebra, and her method was to discard the objects and keep the axioms. An ideal is a subset of a ring that absorbs multiplication by anything in the ring. In 1921 she showed that a single FINITENESS assumption - the ASCENDING chain condition, which forbids any strictly increasing chain of ideals from running forever - forces decomposition results that earlier mathematicians had proved case by case. GROUPS, RINGS, FIELDS, and MODULES stopped being separate subjects and became one, linked by the HOMOMORPHISM, a map that preserves structure rather than content.

SPELL Spell: HOMOMORPHISM

HOMOMORPHISM

SPELL Spell: FINITENESS

FINITENESS

KNOWN What is the chain condition called that forbids any strictly increasing chain of ideals from running on forever?

ASCENDING

SEMI_OPEN Name two of the algebraic structures her axiomatic method treated alike.

NEEDS 2 OF GROUPS / RINGS / FIELDS / MODULES

OPEN Her method threw away the objects and kept only the rules they obey. What is one question you would ask her about working that way?

Open response - use practitioner judgment

THE SECOND THEOREM

The 1918 paper contains two theorems, and the second is the stranger one. The first covers global symmetries, transformations applied identically everywhere. The second covers GAUGE symmetry, whose parameter can be chosen freely at every separate point, as in electromagnetism, and general COVARIANCE, the freedom of general relativity to be written in any coordinate system at all. Local symmetry buys less than it appears to: instead of independent conservation laws it yields identities among the field equations themselves, the contracted BIANCHI identities in gravity. That is why energy in general relativity refuses to behave the way energy behaves elsewhere.

SPELL Spell: COVARIANCE

COVARIANCE

SPELL Spell: BIANCHI

BIANCHI

KNOWN What is a symmetry called when its parameter can be chosen freely and independently at every separate point?

GAUGE

OPEN Local symmetry buys less than it seems to. What is something that looked like a rule and turned out to be an accounting trick?

Open response - use practitioner judgment

MATH In four-dimensional spacetime, the Einstein field equations have 10 independent components, and the contracted Bianchi identities impose 4 constraints among them. How many independent equations are left?

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THE NAME ON THE COVER

The faculty relented in 1919; an unofficial PROFESSORSHIP and a small STIPEND followed in 1922. Her lectures reached print through van der Waerden's *Moderne Algebra* in 1930, carrying her ideas to a generation that never saw her name on the cover. EPONYMY, the habit of hanging a discoverer's name onto a result, is a poor memory system: the label survives, the person evaporates. In 1933 the civil service law expelled her, and she taught at Bryn Mawr until her death two years later at fifty-three. Einstein wrote to the *New York Times* calling her the most significant creative mathematical genius since the higher education of women began.

SPELL Spell: EPONYMY

EPONYMY

SPELL Spell: PROFESSORSHIP

PROFESSORSHIP

SPELL Spell: STIPEND

STIPEND

OPEN Her ideas reached a generation that never saw her name on the cover. Whose work do you use every day without knowing who did it?

Open response - use practitioner judgment

WHY ANYTHING STAYS PUT

You lean on her result whenever you trust that something stays put. A skater pulling her arms inward spins faster because space looks the same in every direction, and that rotational symmetry hands her a fixed ANGULAR momentum. Charge does not quietly drain out of your phone's circuits because the gauge symmetry of ELECTROMAGNETISM forbids it. Where the symmetry is missing, so is the guarantee: light from distant galaxies arrives stretched toward the red, a REDSHIFT, having lost energy on the way, because an expanding universe offers no time-translation symmetry to conserve it. The UNBREAKABLE laws are conditional, and the condition was proved by a woman whose name you were probably never given.

SPELL Spell: **ELECTROMAGNETISM**

ELECTROMAGNETISM

SPELL Spell: **UNBREAKABLE**

UNBREAKABLE

KNOWN What kind of momentum does a skater keep fixed when she pulls her arms in and spins faster?

ANGULAR

KNOWN What is the term for light from distant galaxies arriving stretched toward longer wavelengths, having lost energy crossing an expanding universe?

REDSHIFT

OPEN A skater spins faster the moment she pulls her arms in. When have you felt something speed up just by pulling in?

Open response - use practitioner judgment

MATH A skater conserves angular momentum, so cutting her moment of inertia to a third multiplies her spin rate by three. Suppose she turns at 2 rotations per second with her arms out. How many rotations per second does she make once her arms are in?

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